

# UNDERSTANDING 2D6 MATH

I need to understand the math behind 2d6 rolling and so I decided to write it up in a way that does not require advanced math.

## *The distribution of 2d6*

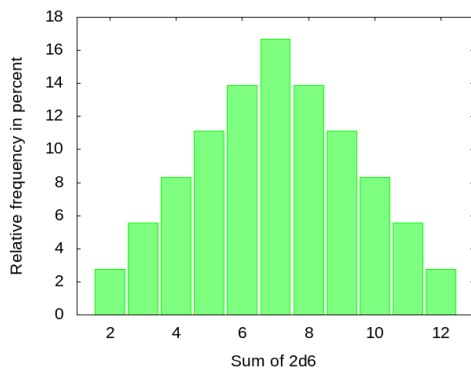
Let's start with a table with all the possible results of the two dice and their sums.

|   |   |   |   |    |    |    |   |
|---|---|---|---|----|----|----|---|
|   | 1 | 2 | 3 | 4  | 5  | 6  | 7 |
| 1 | 2 | 3 | 4 | 5  | 6  | 7  |   |
| 2 | 3 | 4 | 5 | 6  | 7  | 8  |   |
| 3 | 4 | 5 | 6 | 7  | 8  | 9  |   |
| 4 | 5 | 6 | 7 | 8  | 9  | 10 |   |
| 5 | 6 | 7 | 8 | 9  | 10 | 11 |   |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |   |

Now, let's count how often the various sums show up. I like how the numbers form diagonal "lines". Check out 7, for example: from the bottom left to the top right. Anyway, here are the occurrences:

|   |   |   |   |   |   |   |   |    |    |    |
|---|---|---|---|---|---|---|---|----|----|----|
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 2 | 3 | 4 | 5 | 6 | 5 | 4 | 3  | 2  | 1  |

And this is the histogram:



# The odds of reaching a target number

What are the odds of rolling a number or more? We can simply sum the occurrences of all the results that satisfy our requirements and compare that sum to total number of possible rolls:  $6 \times 6 = 36$ .

This is the same table as before. It shows the result of one person rolling 2d6, and the occurrences of each result. Thus, to compute the odds of rolling a 9 or more simply means adding up the highlighted numbers: 6, thus the chances of rolling a 10 or higher is 6:36, i.e. 1:6 or about 17%.

|   |   |   |   |   |   |   |   |    |    |    |
|---|---|---|---|---|---|---|---|----|----|----|
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 2 | 3 | 4 | 5 | 6 | 5 | 4 | 3  | 2  | 1  |

In Traveller, the target number is usually an 8, and people add their difficulty modifier to it. For our purposes, we just subtract from the target number. Thus, to determine the odds of rolling a 7 with 2d6 we add up the highlighted numbers: 21:36, i.e. 7/12 or about 58%.

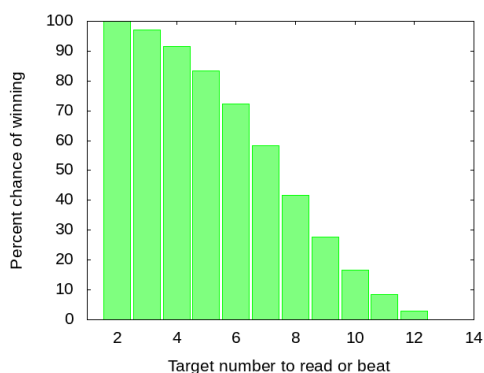
|   |   |   |   |   |   |   |   |    |    |    |
|---|---|---|---|---|---|---|---|----|----|----|
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 2 | 3 | 4 | 5 | 6 | 5 | 4 | 3  | 2  | 1  |

Actually, we can compute this for all the results and simply list our chances of reaching a target number using 2d6.

|      |     |     |     |     |     |     |     |     |    |    |    |
|------|-----|-----|-----|-----|-----|-----|-----|-----|----|----|----|
| 2    | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11 | 12 | 13 |
| 100% | 97% | 92% | 83% | 72% | 58% | 42% | 28% | 17% | 8% | 3% | 0% |

You can verify this on anydice.com. If you enter `loop N over {2..13}{output 2d6 >= N named "Rolling [N] or higher"}` and click on the *Calculate* button, you get two results for each target number: 0 are your chances of not making it, 1 are your chances of making it.

And this is the visualisation of the numbers above. As the target number goes up, your chances go down.



# The odds of beating an opponent

What are the odds of beating your opponent, if you both roll +0? Let's do another table. This time we multiply the numbers from the table above with how often they occur. If we're comparing the result of 4 vs. 6 for example, we'll record  $3 \times 5 = 15$ . A result 4 appears  $3 \times$  and a result of 6 appears  $5 \times$ , so together they appear  $15 \times$ . Once we have these numbers, we add up the numbers where one roll beats the other and compare the sum to the total number of possible rolls:  $6 \times 6 \times 6 \times 6 = 1296$ .

The table shows the results of two people rolling 2d6, in rows and columns, respectively. Adding up the highlighted numbers: 576, thus the chances of beating somebody else is 576:1296 or about 44%.

|    | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|----|---|----|----|----|----|----|----|----|----|----|----|
| 2  | 1 | 2  | 3  | 4  | 5  | 6  | 5  | 4  | 3  | 2  | 1  |
| 3  | 2 | 4  | 6  | 8  | 10 | 12 | 10 | 8  | 6  | 4  | 2  |
| 4  | 3 | 6  | 9  | 12 | 15 | 18 | 15 | 12 | 9  | 6  | 3  |
| 5  | 4 | 8  | 12 | 16 | 20 | 24 | 20 | 16 | 12 | 8  | 4  |
| 6  | 5 | 10 | 15 | 20 | 25 | 30 | 25 | 20 | 15 | 10 | 5  |
| 7  | 6 | 12 | 18 | 24 | 30 | 36 | 30 | 24 | 18 | 12 | 6  |
| 8  | 5 | 10 | 15 | 20 | 25 | 30 | 25 | 20 | 15 | 10 | 5  |
| 9  | 4 | 8  | 12 | 16 | 20 | 24 | 20 | 16 | 12 | 8  | 4  |
| 10 | 3 | 6  | 9  | 12 | 15 | 18 | 15 | 12 | 9  | 6  | 3  |
| 11 | 2 | 4  | 6  | 8  | 10 | 12 | 10 | 8  | 6  | 4  | 2  |
| 12 | 1 | 2  | 3  | 4  | 5  | 6  | 5  | 4  | 3  | 2  | 1  |

What if you have a +1 advantage? Just add one more diagonal:  $576 + 146 = 722$ , so your odds are now 722:1296 or about 56%.

|    | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|----|---|----|----|----|----|----|----|----|----|----|----|
| 2  | 1 | 2  | 3  | 4  | 5  | 6  | 5  | 4  | 3  | 2  | 1  |
| 3  | 2 | 4  | 6  | 8  | 10 | 12 | 10 | 8  | 6  | 4  | 2  |
| 4  | 3 | 6  | 9  | 12 | 15 | 18 | 15 | 12 | 9  | 6  | 3  |
| 5  | 4 | 8  | 12 | 16 | 20 | 24 | 20 | 16 | 12 | 8  | 4  |
| 6  | 5 | 10 | 15 | 20 | 25 | 30 | 25 | 20 | 15 | 10 | 5  |
| 7  | 6 | 12 | 18 | 24 | 30 | 36 | 30 | 24 | 18 | 12 | 6  |
| 8  | 5 | 10 | 15 | 20 | 25 | 30 | 25 | 20 | 15 | 10 | 5  |
| 9  | 4 | 8  | 12 | 16 | 20 | 24 | 20 | 16 | 12 | 8  | 4  |
| 10 | 3 | 6  | 9  | 12 | 15 | 18 | 15 | 12 | 9  | 6  | 3  |
| 11 | 2 | 4  | 6  | 8  | 10 | 12 | 10 | 8  | 6  | 4  | 2  |
| 12 | 1 | 2  | 3  | 4  | 5  | 6  | 5  | 4  | 3  | 2  | 1  |

OK, so now we know how to compute the odds of winning opposed 2d6 rolls taking into account an advantage: find the appropriate triangle starting at the bottom-left and sum up all the numbers in that triangle, and compare that to the total number (1296).

Here's the result:

|     |     |     |     |     |     |     |     |     |       |       |      |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|-------|------|
| +0  | +1  | +2  | +3  | +4  | +5  | +6  | +7  | +8  | +9    | +10   | +11  |
| 44% | 56% | 66% | 76% | 84% | 90% | 95% | 97% | 99% | 99.6% | 99.9% | 100% |

What about the reverse? When you have a penalty, the situation is reversed: you can look up the chance of your opponent winning with your penalty being their bonus. Thus, if your penalty is -1, their chance of winning is 56%. Remember, however, that nobody wins when you both get the same result. Therefore, your chance of winning with a penalty is worse: it's  $100-66=34\%$ .

You can verify this on anydice.com. If you enter `loop N over {-11..11}{output 2d6-2d6<N} named "with modifier [N]"` and click on the *Calculate* button, you get two results for each target number: 0 are your chances of not making it, 1 are your chances of making it.

And this is the visualisation of the numbers above. As the advantage goes up, your chances go up.

